Robust Fitting of Super-Helices to Parametric Curves

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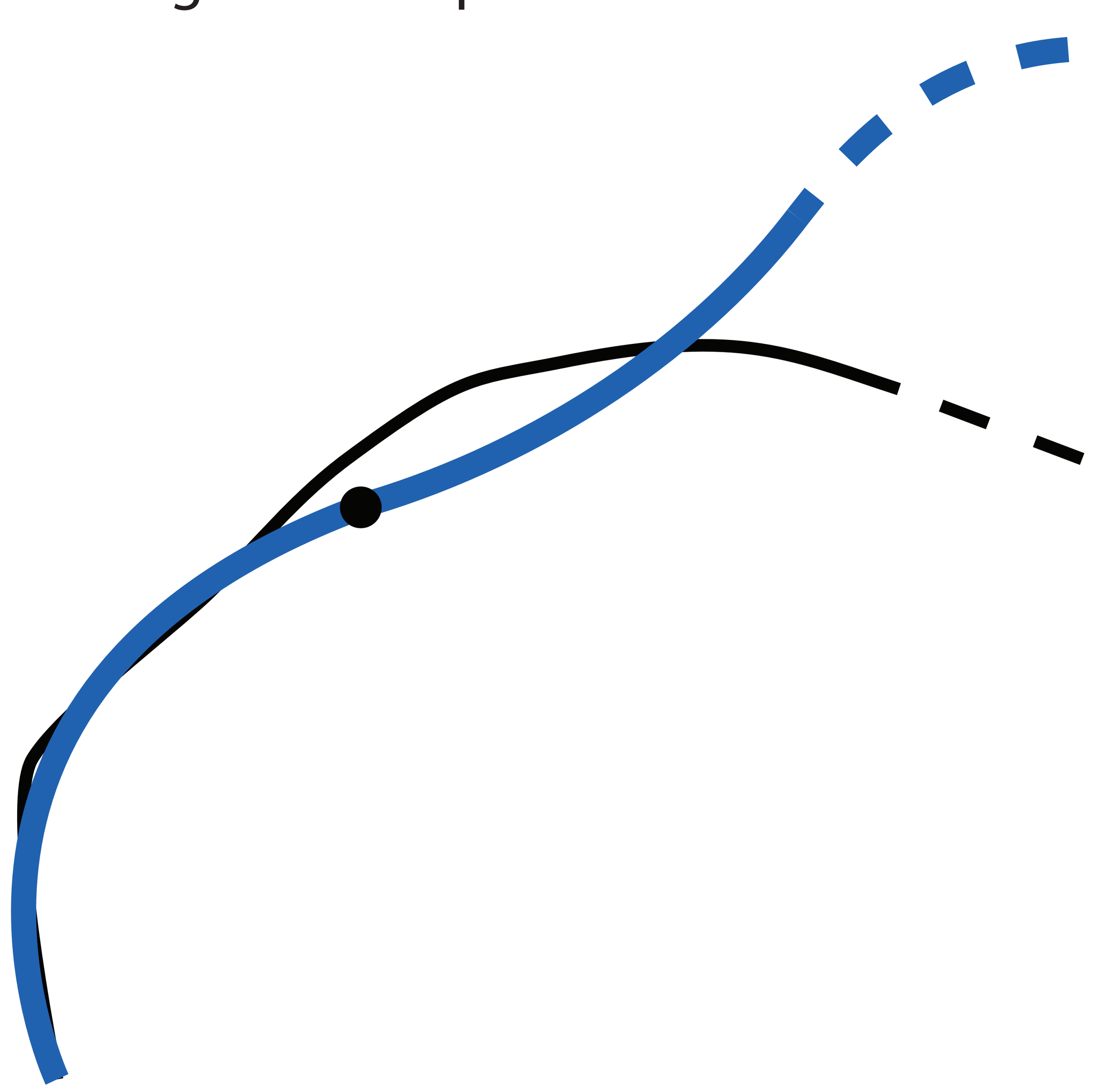
Introduction

Recently Bertails *et al.* [1] proposed using Cosserat curves with piecewise constant curvature - so called Super-Helices (SH) - for dynamic simulations of hair. While this has proven benificial in several aspects, it comes at the price of having to represent curves (or hair strands) in an unfamiliar and largely unintuitive parameter-space of curvatures and twists. Consequently the modeling and styling of SH curves is a challenging task, one that has not been addressed in previous work.

Our approach employs a combination of data reduction and error analysis known from mesh decimation algorithms as well as non-linear minimization, in order to fit SH curves to arbitrary parametric curves, in particular NURBS. Consequently, this allows us to take advantage of the large body of existing work on parametric curve modeling.



Error accumulation while naïvely fitting SH curve (blue) to NURBS curve (black).

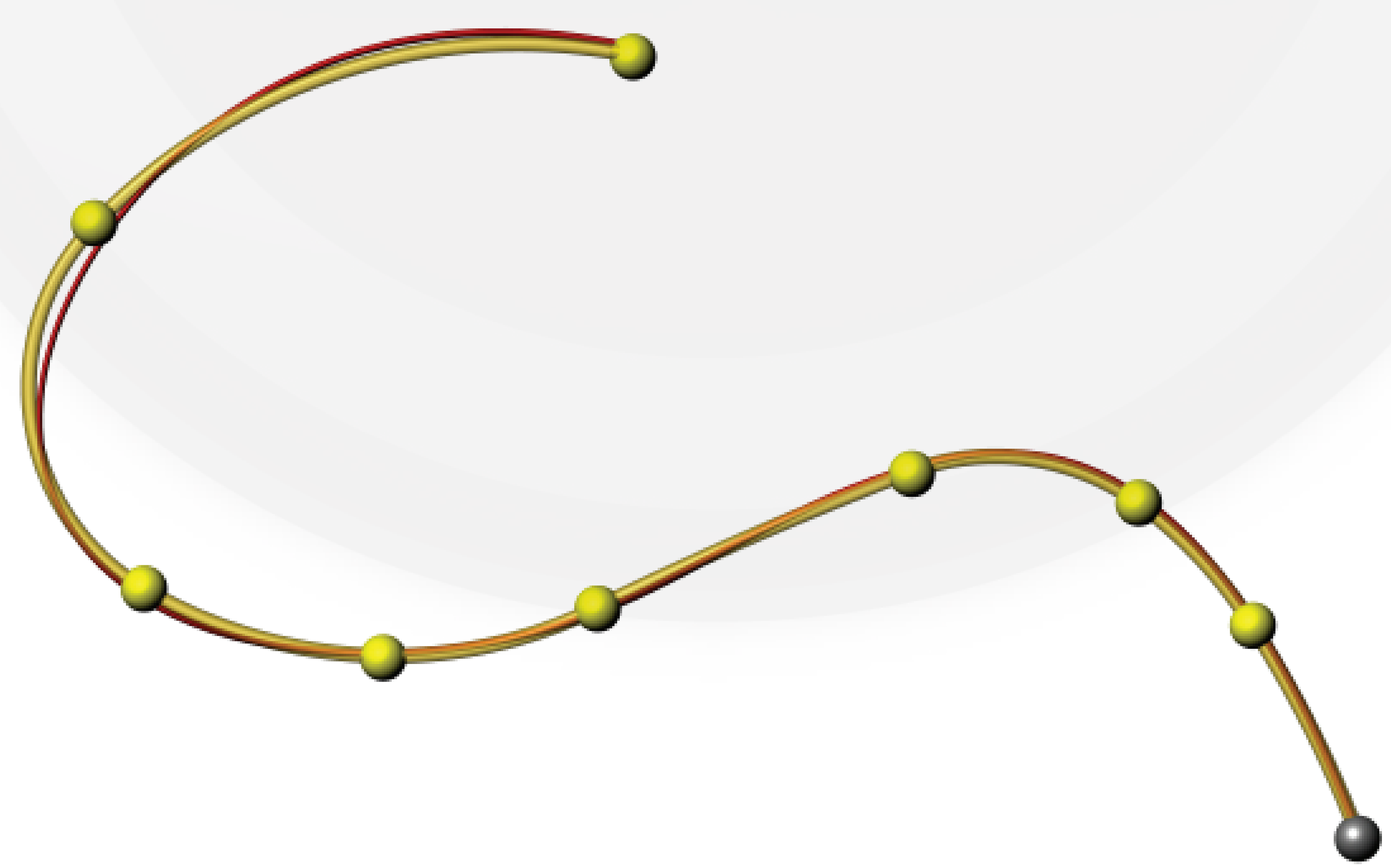
Problem statement

In general terms we wish to fit a SH curve $\mathbf{r}_N(s)$, with N elements having constant curvatures $\mathbf{q}_i=\{q_0,q_1,q_2\}$, to a parametric (i.e. NURBS) curve $\mathbf{Y}(s)$, parameterized in archlength s.

A straightforward root-to-tip, per-segment quasi-Newton minimization of the functional

$$\chi^2(\mathbf{q}) = \Sigma(\mathbf{Y}(s) - \mathbf{r}(s))^2$$

suffers from severe stability problems due to the way error accumulates throughout the SH. On the other hand, exploiting the coupling between a segment and its predecessors and performing global fitting introduces a vast parameter space with an abundance of local, visually appalling, minima. Working with a coupled Hessian also requires the solution of a $3N$x$3N$, *dense* matrix for each iteration, which further restricts the resolution of the Super-Helix as the number of iterations likely will be high.



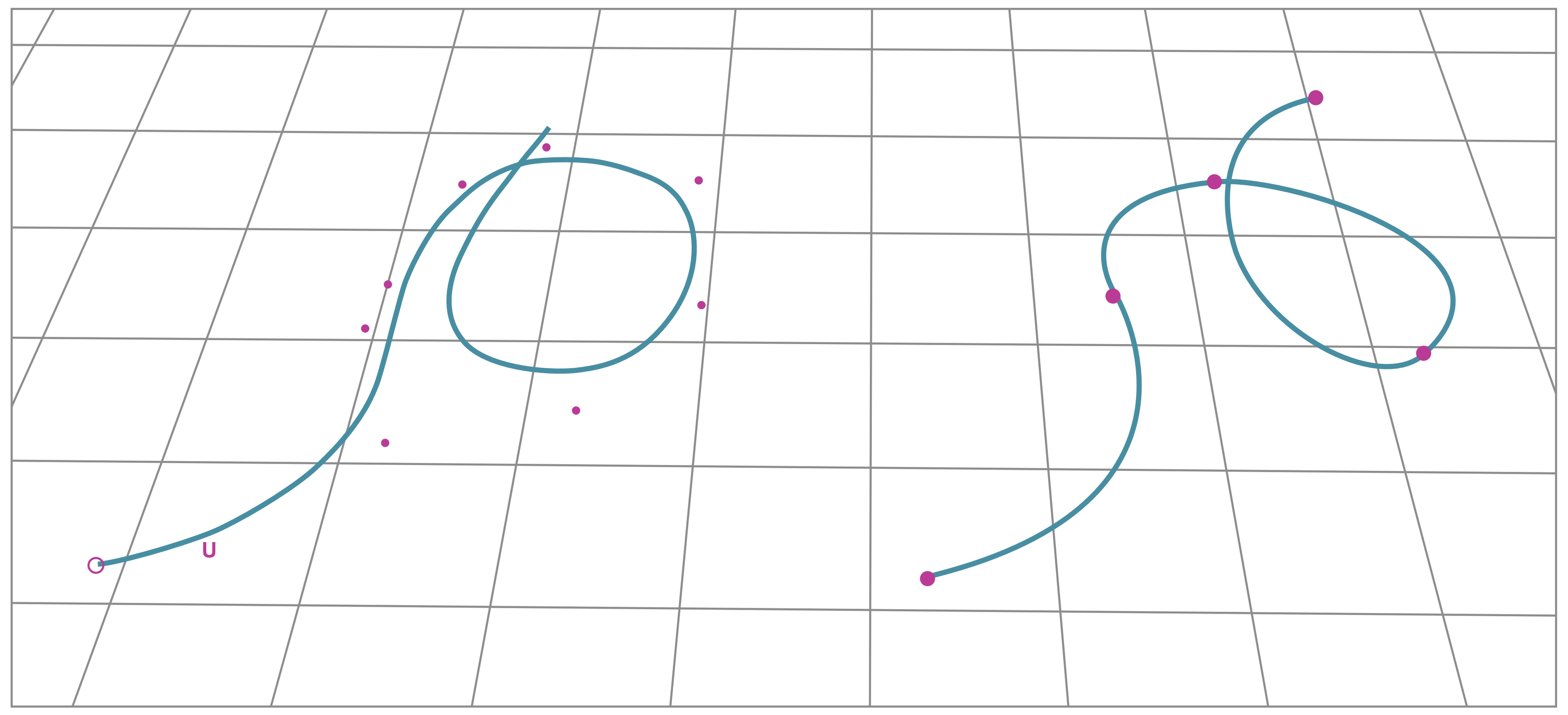
Super-Helix (yellow) fitted to NURBS curve (red).

Our method

1) Perform a **high-resolution fit** based on the Frenet-Serret curvature of the NURBS curve. Each SH segment is fitted locally using a classic minimization technique on error metrics such as endpoint position and orientation.

2) Perform **global error minimization** on the SH. The Frenet-Serret analysis in step 1 effeciently produces an initial SH curve with enough elements to limit the accumulated error and accurately fit to the NURBS curve. This is essential, since it reduces the risk of getting stuck in an unsuitable local minima as well as improves the rate of convergence. The non-linear optimization is performed using a modified Levenberg-Marquardt algorithm with a Preconditioned Conjugate Gradient solver.

3) **Reduce the high-resolution SH** to a resolution feasible for simulation by iteratively removing the segment which introduces the least amount of error. Its arclength is distributed over its neighbors and the SH is relaxed using non-linear optimization. This procedure is repeated until sufficiently many segments have been removed.



Super-Helix (right) fitted to NURBS curve (left).

[1] F. Bertails, B. Audoly, M.-P. Cani, B. Querleux, F. Leroy, and J.-L. Lévêque. Super-Helices for predicting the dynamics of natural hair. In *ACM Transactions on Graphics (Proceedings of the SIGGRAPH conference)*, August 2006.